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An Introduction to Models and Probability Concepts

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According to the Operations Research Society of America, "Operations research is concerned with scientifically deciding how to best design and operate man-machine systems, usually under conditions requiring the allocation of scarce resources." This publication is the first in a series describing operations research (OR) techniques that can help forest products managers solve complex problems. It will introduce basic concepts of models and probability.

Models

No matter how OR is defined, the construction and use of models is at its core. Models are representations of real systems. They can be iconic (made to look like the real system), abstract, or somewhere in between.

Iconic models can be full-scale, scaled-down, or scaled-up in size. Sawmill headrig control simulators are full-scale models. A model of the solar system is a scaled-down model, and a teaching model of a wood cell or a water molecule is a scaled-up model. Models can be made of the same material as the system they represent, such as the headrig control simulator, or they can be made of different materials, such as a plastic model of the solar system.

On the other end of the model spectrum are abstract mathematical models (Figure 1). OR professionals often use mathematical models to make simplified representations of complex systems.

Regardless of the type of model used, modeling includes the following steps:

1. Defining the problem and gathering data
2. Constructing a model of the system
3. Deriving a solution
4. Testing the model and solution (Is the model valid; that is, does it do what it is designed to do?)
5. Implementing the solution

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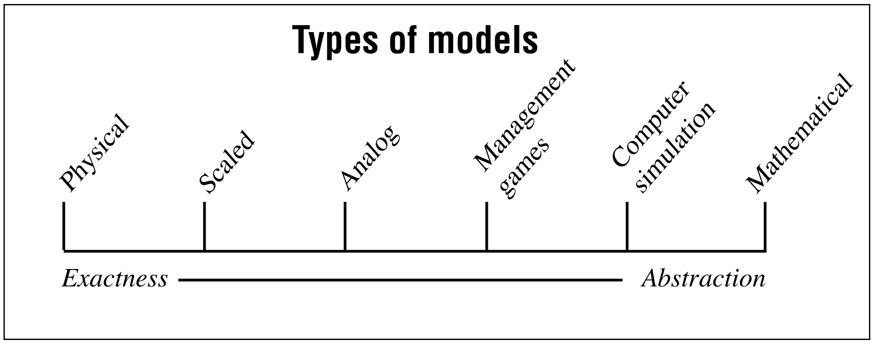


Figure 1.—Models range from physical to abstract (Source: Shannon, R.E. *Systems Simulation: The Art and Science)*.

Why use models?

Why not experiment with the actual system? If the actual system is simple enough to manipulate safely, then OR techniques often are not necessary. For example, managers of a sawmill might wish to extend the hours for loading trucks from 5:00 p.m. until midnight. Since the shipping shed is adjacent to the planer mill, they decide to use the planer mill employees and possibly one or two overtime employees from shipping to extend the loading hours for a trial period of 2 to 3 weeks. At the end of this time, they'll have a good idea of how the extended shipping hours affected the rest of the sawmill operations.

An OR professional also could model this problem, perhaps using a scheduling linear program or a simulation model to examine the effect of extending the shipping hours. However, if the real system can be manipulated without causing too much disruption, it would be preferable to do so rather than to have an OR professional model it. Often, it can take weeks or months just to collect enough data to realistically model a system. In this case, it probably would be less expensive to manipulate the real system.

In other cases, manipulating the real system is neither simpler nor less expensive than building a model. For example, very long time periods may be involved, or management may wish to examine substitutes for the real system. Also, real systems often are too complex to experiment with directly. The actual experimentation may be infeasible, disruptive, or too expensive. There are several ways to study a system without experimenting with the real system (Figure 2).

Why use models?

- Real systems may be too complex.
- Models may be less expensive.
- Models can test options without disrupting the real system.
- Modeling provides new insights into the real system.

For example, meteorologists build models to study the weather because they can't experiment with the weather itself. One problem is repeatability. It's impossible to set up repeatable experiments with systems over which the researcher has no control, such as the weather. Meteorologists can observe but not test the system. With a model, however, they can test their theories. They can run the model under different conditions and observe the weather to see whether the model is a good enough (although simplified) representation of the real system. Using the same methods, an experiment using the model is repeatable.

In the context of forest products manufacturing, a moulding and millwork manufacturer may want to expand. Management wishes to see whether it would be cost-effective to build an addition with new computer numerically controlled (CNC) equipment adjacent to the existing facility. The manufacturer is unlikely to build the addition, purchase new CNC equipment, run the operation for a year, and then decide whether to keep the addition based on this experiment. It would be too expensive and probably would disrupt the current manufacturing operation. A simulation of the addition with new equipment, including details of how it would interact with the established manufacturing facility, would allow the owner to test how the new facility would work. He or she could run a test

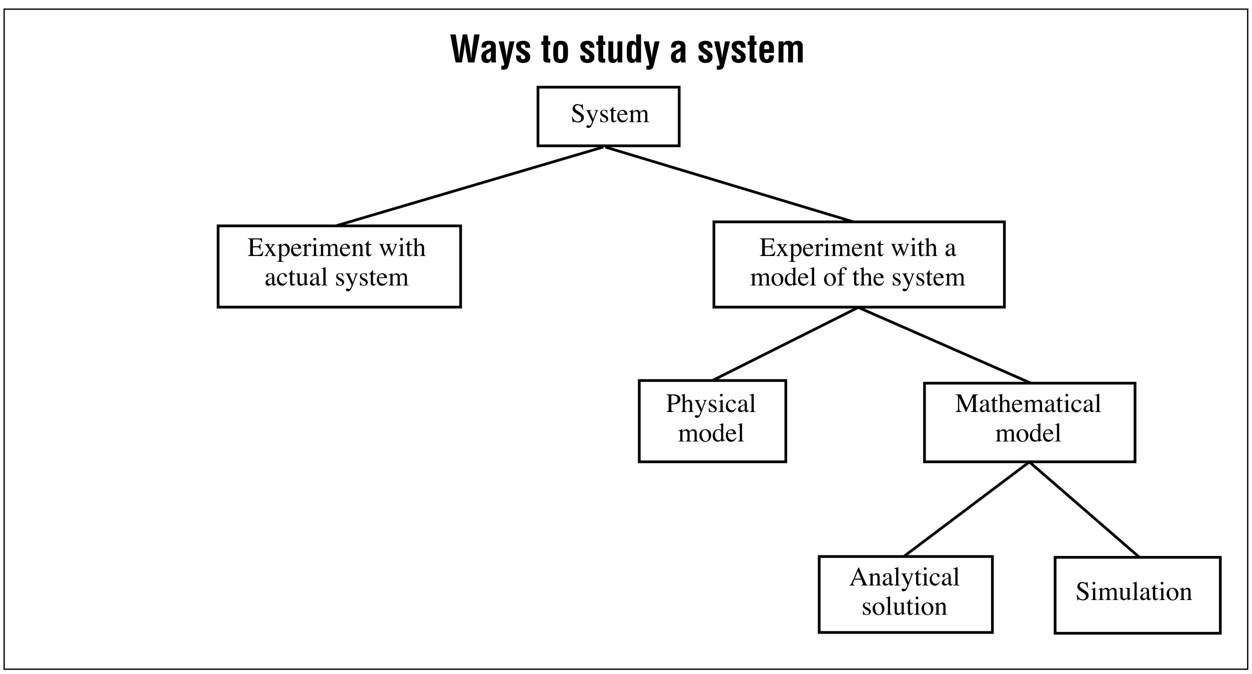


Figure 2.—Operations research experts can study actual systems, but usually use models instead (Source: Law, A.M., and W.D. Kelton. *Simulation Modeling and Analysis*).

of different periods of time (e.g., months or years) to examine the impact of the new facility on the entire operation.

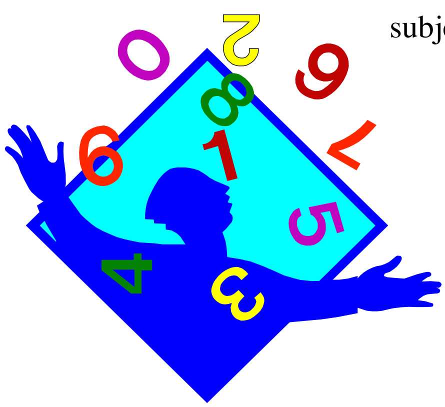
Other uses for models are for instruction and training, such as a sawmill headrig simulator or NASA's full-scale mockup of space vehicles for astronaut training.

An important, but often overlooked, benefit to modeling is the insight one gets from the process. Often, modelers learn a lot about a system during problem definition and data gathering. An experimenter must organize his or her thoughts to model a system successfully, and this thought process often reveals previously overlooked insights. Information gained from these new insights can lead to problem resolution even before the model is completed.

Since modeÑs are simàlified representations of real systems, they can't be proven. It's unlikely that two OR professionals working on the same problem independently would create the same model. If the problem involved the use of simulation, one modeler might write a program using Fortran, another might use Basic, and another might use an off-the-shelf simulation language. However, one model might be just as good as the other. The usefulness of any model depends on how well it addresses the problem.

Modeling can be seen as an "art" as much as a science. Ravindran et al. (1987) contrast the scientific method with the modeling process. Using the scientific method, one makes observations, develops a hypothesis, experimentally tests the hypothesis, and revises and retests it if necessary until a verified hypothesis or theory is obtained. Unlike models, theories are testable and independently verifiable. Theories are discovered, while models are invented.

An important step in model creation is validation. A model is valid if it does what it was intended to do. The model creator may believe the model is valid, while other users may not. Thus, validation is not as rigid a term as verification or proof and is somewhat subjective.



What makes a good model?

A simple model is better than a complex one as long as it works as well. A model only needs to perform its intended function to be valid. A model should be easy to understand.

It's important to use the most relevant OR tool when constructing a model. A modeler should not try to shape the problem to fit a particular OR method. For example, a linear programming (LP) expert may try to use LP on a problem where there is no optimal solution. Instead, modelers should study the problem and choose the most appropriate OR tool.

For complicated systems, users need to remember that models are only simplified representations. If a user mistakenly considers a complicated model to be correct, he or she may disregard further study of the real system. Modelers and users of models never should rely only on a model's output and ignore the real system being modeled.

A good model should be easy to modify and update. New information from the real system can be incorporated easily into a well-planned model. A good model usually starts out simple and becomes more complex as the modeler attempts to expand it enough to give meaningful answers.

Good models are:

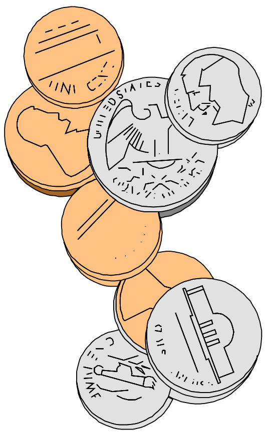
- As simple as possible
- Easy to understand
- Relevant to the problem
- Easy to modify and update

Probability theory

The relationship between models and probability

Models can be *deterministic* or *stochastic*. A deterministic model contains no random (probabilistic) components. The output is determined once the set of input quantities and relationships in the model have been specified. Stochastic models, on the other hand, have one or more random input components. For this kind of model, probability and statistics are important, because we use them to measure uncertainty. Many, if not most, OR models are stochastic.

Probability provides the basis for all statistical inference by allowing the experimenter to assess the chance of various outcomes. It is fundamental in analyzing decision-making problems involving incomplete information. In other publications in this series, you'll be introduced to simulation and decision theory. Both of these OR tools depend heavily on probability theory. A brief review of probability theory will help you to better understand how these OR techniques operate.



What is probability?

The probability of an event is the relative frequency at which it occurs when the identical situation is repeated a large number of times (Equation 1).

$$\textit{Eq. 1}\quad \text{Probability (Pr) of an event} = \frac{\text{Number of times the event occurs}}{\text{Number of times the situation is repeated}}$$

It's easy to calculate the probability of an event if all possible outcomes are known (Equation 2).

$$\textit{Eq. 2}\quad \text{Pr of an event} = \frac{\text{Number of times the event can occur}}{\text{Total number of all events that can occur}}$$

For example, the probability of a head occurring when tossing a two-sided coin is:

Pr of a head = 1/2
(There are only two possible outcomes for every toss.)

Or, the probability of getting an even-numbered side when tossing a six-sided die is:

Pr of even-numbered side = 3/6 or 1/2
(Three of the six sides are even numbered.)

Other examples:

Pr of drawing a club from a deck of cards = 13/52 or 1/4
(There are 13 clubs out of 52 cards.)

Pr of drawing a face card = 12/52 or 3/13
(There are 12 face cards out of 52 cards.)

Sometimes, however, not all events that can occur are known. In this case, an experiment can be conducted to estimate the probability of the event. As the numerator (top number in the equation) and denominator (bottom number) get sufficiently large, an experiment will come close to the true probability of the event.

A probability always is between 0 and 1 because the numerator can be neither negative nor larger than the denominator.

Probabilities for compound events

Compound events refer to situations where multiple events can occur. You may want to consider the probability of more than one of them occurring.

> ***Addition laws. . .***
>
> "The probability that any of a set of possible events will occur."

Addition laws

Addition laws are used to assess the probability that any of a set of possible events will occur (e.g., A *or* B *or* C). The simplest form is the addition law for *mutually exclusive events* (events that cannot occur simultaneously), which is given in Equation 3.

Eq. 3 Pr [A or B or C] = Pr[A] + Pr[B] + Pr[C]

For example, suppose the number of logs that can be sawn on a sawmill headrig during the first hour of mill operation ranges from 0 (the mill is down for some reason) to 250 (the best the mill has ever done). By observation and collection of a large amount of data, the following estimations for probabilities of sawing logs in the first hour were calculated (Table 1).

Table 1.—Probabilities of sawing logs in the first hour.

Number of logs sawn during first hour of operation	Probability
0 (mill down)	0.1
1–50	0.05
51–100	0.1
101–150	0.3
151–200	0.43
201–250	0.02

Pr of sawing at least 101 logs = Pr[101–150] + Pr[151–200] + Pr[201–250]

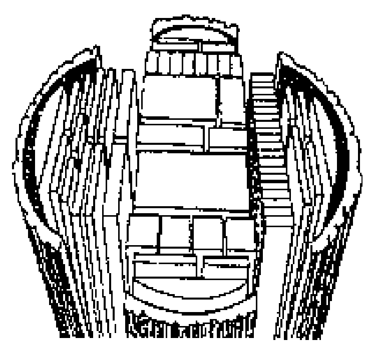
= 0.3 + 0.43 + 0.02 = 0.75

Pr of not sawing at least 101 logs = Pr[0] + Pr[1–50] + Pr[51–100]

= 0.1 + 0.05 + 0.1 = 0.25

Pr of sawing at least 101 or not sawing at least 101 logs = 0.75 + 0.25

= 1.0



When events are not mutually exclusive, *joint probability* (the probability that both events might occur simultaneously) must be subtracted from the sum of the probabilities that each individual event will occur to avoid double counting of possibilities (Equation 4).

Eq. 4 Pr[A or B] = Pr[A] + Pr[B] - Pr[A and B]

For example, for a deck of cards, Pr of drawing an ace is $^{4}/_{52}$ (there are 4 aces out of 52 cards), and Pr of drawing a heart is $^{13}/_{52}$ (there are 13 hearts out of 52 cards). The probability of drawing an ace of hearts is $^{1}/_{52}$ (there is only one ace of hearts in a deck). Thus, the probability of drawing an ace *or* a heart is:

Pr[ace or heart] = Pr[ace] + Pr[heart] - Pr[ace + heart]

= 4/52 + 13/52 - 1/52

= 16/52

Multiplication laws. . .

"The probability that more than one of a set of possible events will occur."

Multiplication law for independent events

Multiplication laws are used to look at the probability that more than one of a set of possible events will occur (e.g., both A *and* B). The simplest multiplication law applies to situations where the events operate independently, that is, they don't have any effect on each other (Equation 5). An example is the probability of drawing an ace of hearts.

Eq. 5 Pr[A and B] = Pr[A] x Pr[B]

For a deck of cards:

Pr[ace and heart] = Pr[ace] x Pr[heart]

= 4/52 x 13/52

= 1/52

Let's say a contractor wishes to order lumber from a wholesale lumber supplier. The supplier has three phone lines (three different numbers that operate *independently* of each other). The probability of phone 1 being busy is 0.80, the probability of phone 2 being busy is 0.80, and the probability of phone 3 being busy is 0.80. The contractor calls to order 50 thousand board feet (MBF) of pine. What is the probability that the phone lines will be busy?

Pr[phone 1 and 2 busy] = Pr[phone 1 busy] x Pr[phone 2 busy]

= 0.80 x 0.80 = 0.64

Pr[phone 1, 2, and 3 busy] = Pr[1 busy] x Pr[2 busy] x Pr[3 busy]

= 0.80 x 0.80 x 0.80

= 0.512

Pr of getting through (at least one phone line not busy) = 1 - Pr[all busy]

= 1 - 0.512

= 0.488

Joint probability

Joint probability refers to obtaining probabilities when several events can occur simultaneously. Let's say we're going to choose a student, by lottery, from a group of 200 equally gifted students. We want to know the chances of choosing a particular student type based on four demographic characteristics. The demographics of the students are illustrated in a joint probability table (Table 2), which allows you to calculate the probability of simultaneously occurring events.

> ***Joint probability. . .***
>
> "The probability of several events occurring simultaneously."

Table 2.—Joint probability table of gifted students.

	Caucasian (c)	**Minority (m)**	**Total**
Male (ma)	55	40	95
Female (f)	65	40	105
Total	120	80	200

Probability of choosing a Caucasian male:

Pr[ma and c] = 55/200 = 0.275

Probability of choosing a minority male:

Pr[ma and m] = 40/200 = 0.20

Probability of choosing a Caucasian female:

Pr[f and c] = 65/200 = 0.325

Probability of choosing a minority female:

Pr[f and m] = 40/200 = 0.20

Probability of choosing a Caucasian:

Pr[c] = 120/200 = 0.60

Probability of choosing a minority is:

Pr[m] = 80/200 = 0.40

> **_Conditional probability. . ._**
>
> "The probability that an event will occur given that some other event has occurred."

Conditional probability

The occurrence of one event can affect the probability of another event. Probability values obtained under the stipulation that some events have occurred or will occur are *conditional probabilities*. We can compute conditional probability from joint probability.

The conditional probability of A given B is the proportion of times that A occurs out of all the times that B occurs (Equation 6). Note that in conditional equations the vertical bar (|) is the symbol for "given."

Eq. 6 $$\Pr[A \mid B] = \frac{\Pr[A \text{ and } B]}{\Pr[B]}$$

Similarly:

$$\Pr[B \mid A] = \frac{\Pr[A \text{ and } B]}{\Pr[A]}$$

Referring to our student example above, the conditional probability that the student chosen is a male, given he is a minority, is:

$$\Pr[ma \mid m] = \frac{\Pr[ma \text{ and } m]}{\Pr[m]} = \frac{0.2}{0.4} = 0.5$$

What is the probability that the first person selected is a male (ma_1), the second person selected is a male (ma_2), and the third person selected is a male (ma_3)?

$$\Pr[ma_1 \text{ and } ma_2 \text{ and } ma_3] = \Pr[ma_1] \times \Pr[ma_2 \mid ma_1] \times \Pr[ma_3 \mid ma_1 \text{ and } ma_2]$$

So:

$\Pr[ma_1] = 95/200$

$\Pr[ma_2 \mid ma_1] = 94/199$
(If $[ma_1]$ happens, there will be 94 males left out of 199 students.)

$\Pr[ma_3 \mid ma_1 \text{ and } ma_2] = 93/198$
(If $[ma_1]$ and $[ma_2]$ happen, there will be 93 males remaining out of 198 students.)

Therefore, the probability of choosing 3 males is:

$$\begin{aligned}\Pr[ma_1 \text{ and } ma_2 \text{ and } ma_3] &= (95/200) \times (94/199) \times (93/198)\\ &= (0.475) \times (0.472) \times (0.470)\\ &= 0.105\end{aligned}$$

Independent events

Events are *independent* (the probability of one event is unaffected by the occurrence of the other) when their conditional probabilities equal their respective unconditional probabilities. Let's determine whether the events [king] and [face] in a deck of cards are independent:

Unconditional probability: Pr[king] = 4/52 = 1/13

$$\text{Conditional probability: Pr[king} \mid \text{face card]} = \frac{\text{Pr[king and face]}}{\text{Pr[face]}} = \frac{4/52}{12/52} = 4/12 = 1/3$$

The conditional probability does not equal the unconditional probability, so the events are not independent.

On the other hand, the events ace and heart are independent. To illustrate this:

Unconditional probability: Pr[ace] = 4/52 = 1/13

$$\text{Conditional probability: Pr[ace} \mid \text{heart]} = \frac{1/52}{13/52} = 1/13$$

In this case, the conditional probability and the unconditional probability are equal, so the events are independent.

> ***Independent events. . .***
>
> "Events whose probabilities are not affected by the occurrence of each other."

Multiplication law for events that are not independent

To find the probability of the joint occurrence of two or more events, you usually use the multiplication law. On page 8, we discussed the multiplication law for independent events. Equation 7 is the multiplication law for events that are not independent.

Eq. 7 Pr[A and B] = Pr[A] x Pr[B | A]

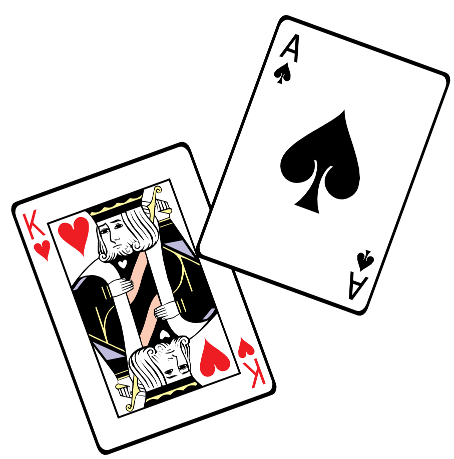
$$\text{Pr[ace and club]} = \text{Pr[ace]} \times \text{Pr[club} \mid \text{ace]}$$

$$= \frac{\text{Pr[ace]} \times \text{Pr[club and ace]}}{\text{Pr[ace]}}$$

$$= \frac{4/52 \times 1/52}{4/52}$$

$$= 1/13 \times 1/4$$

$$= 1/52$$



Probability trees

In Table 2, we used a joint probability table for determining the probability of choosing a particular student type. Joint probability tables can get awkward when dealing with more than two events. A probability tree is one way to deal with this situation.

From our student demographic example, if we look only at determining the probability of choosing a male the first time (ma_1), second time (ma_2), and third time (ma_3), that is, Pr[ma_1 and ma_2 and ma_3], we can use a probability tree to calculate the probability.

We set up marginal probabilities for the first branches, that is, Pr[ma] = $^{95}/_{200}$ and Pr[f] = $^{105}/_{200}$. (The sum of these branches must equal 1). At the tip of each of these branches, we construct branches for another set of probabilities using conditional probabilities based on the first branch. Since there is a third event, another set of branches using conditional probabilities is constructed. The probability of each path is found by multiplying the probabilities along each path. Note that the sum of probabilities for each set of branches must equal 1. Referring to Figure 3:

$$\Pr[ma_1 \text{ and } ma_2 \text{ and } ma_3] = (95/200) \times (94/199) \times (93/198)$$

$$= 0.475 \times 0.472 \times 0.470$$

$$= 0.105$$

You can see that this result is the same as the one we calculated on page 10.

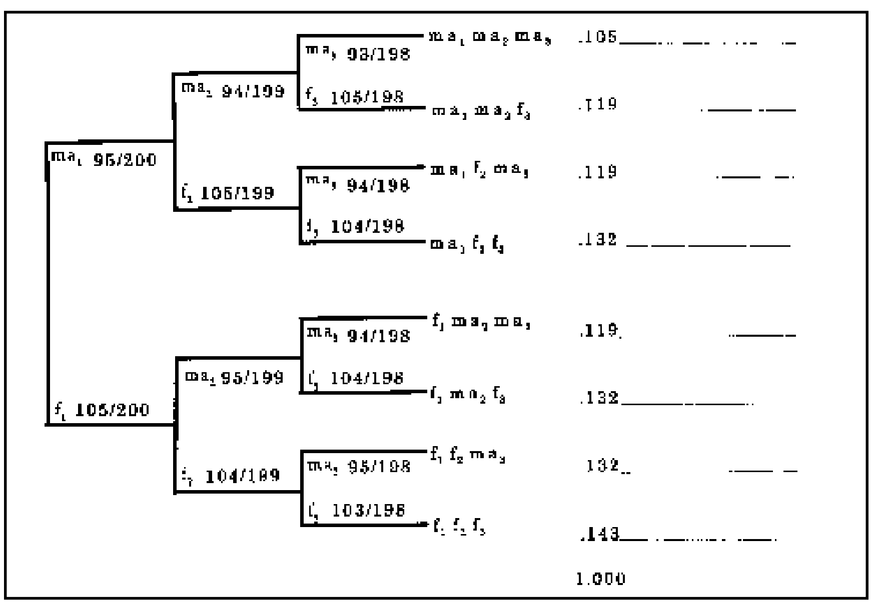


Figure 3.—Probability tree for a random sample of 3 students without replacement from a population of 200 students.

As another example, consider a quality control sampling scheme at a computer numerically controlled (CNC) router manufacturer. Five out of 100 parts from a vendor are defective, but this fact is unknown to the quality control supervisor, who must decide whether to accept or reject a shipment based on a sample of 3 items chosen at random. Let G = good item and D = defective item. The outcome of each of the three sample observations can be represented by a branch. Since one of two complementary (G or D) events can occur for each item observed, there are two branches for each item (Figure 4).

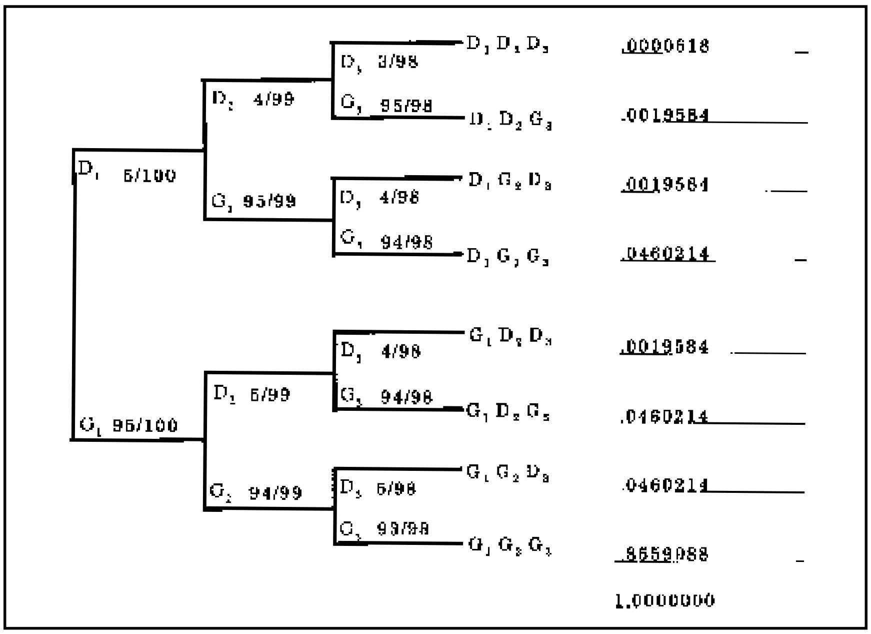


Figure 4.—Probability tree for selecting a random sample of 3 items without replacement from a population of size 100 (Source: Lapin, L.L. *Quantitative Methods for Business Decisions with Cases*).

The results of each selection are not independent because each item is not replaced in the shipment after it is inspected. Therefore, the makeup of the remainder of the shipment changes each time. If D_1 occurs, that is, the first sample item is defective, only 4 of the remaining 99 items can be defective, and the probability that the second item is defective is $4/99$. On the other hand, if G_1 occurs as the first event, then the probability that D_2 occurs is $5/99$. These values are conditional probabilities because the outcome of the second sample depends on the outcome of the first sample and so on.

The joint probabilities for each event can be found by multiplying the probabilities for the branches on the path leading to a particular outcome. The probability that all three items in the sample are defective is:

$$\Pr[D_1 + D_2 + D_3] = \Pr[D_1] \times \Pr[D_2 \mid D_1] \times \Pr[D_3 \mid D_1 \text{ and } D_2]$$

$$= 5/100 \times 4/99 \times 3/98$$

$$= 0.0000618$$

The probability that all three items in the sample are good is:

$$\Pr[G_1 + G_2 + G_3] = 95/100 \times 94/99 \times 93/98$$

$$= 0.8559988$$

All other joint probabilities for each of the possible combinations of three samples are calculated in the same manner.

Based on the above calculations, there is a high probability that the shipment will be accepted. If it is important to reject a shipment (for example, if management will tolerate no more than 1 percent defect), then another sampling scheme should be used.

Let's consider the same problem from the viewpoint of the manufacturer. The quality control supervisor wishes to use sample data to determine whether plant machinery should be adjusted. Unknown to the supervisor, defects occur 5 percent of the time. A probability tree can be constructed for three randomly selected samples of separate items from the production line (Figure 5).

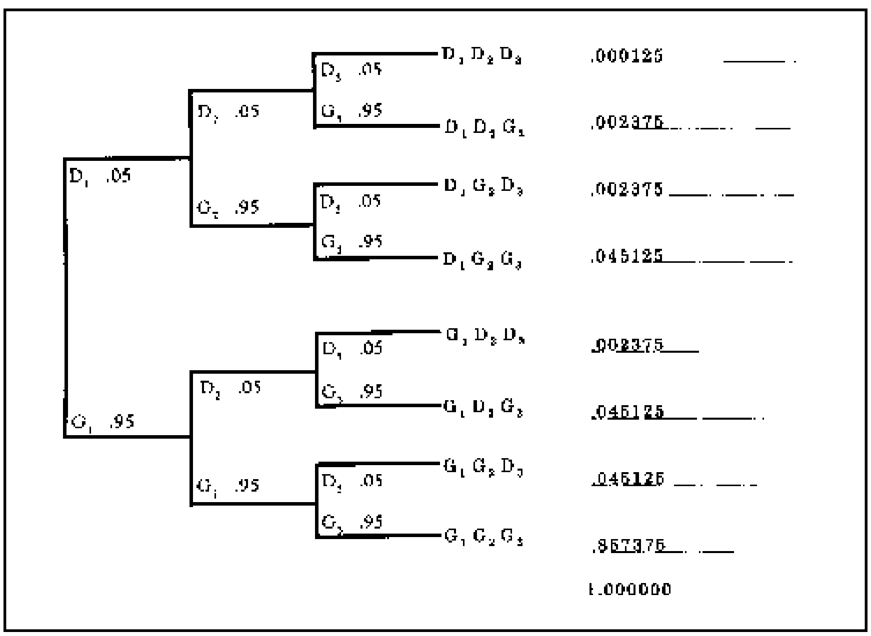


Figure 5.—Probability tree for a random sample of three items from a production process (Source: Lapin, L.L. *Quantitative Methods for Business Decisions with Cases*).

The successive events for any tree path are independent because errors in the process are assumed to be erratic (random). The occurrence of a defect does not influence the occurrence of a defect in the future. The conditional and unconditional probabilities are equal. The probability that all of the samples will be defective is:

$$\begin{aligned} Pr[D_1 + D_2 + D_3] &= Pr[D_1] \text{ x } Pr[D_2] \text{ x } Pr[D_3] \\ &= 0.05 \text{ x } 0.05 \text{ x } 0.05 \\ &= 0.000125 \end{aligned}$$

The probability that all of the samples will be good is:

$$\begin{aligned} Pr[G_1 + G_2 + G_3] &= 0.95 \text{ x } 0.95 \text{ x } 0.95 \\ &= 0.857375 \end{aligned}$$

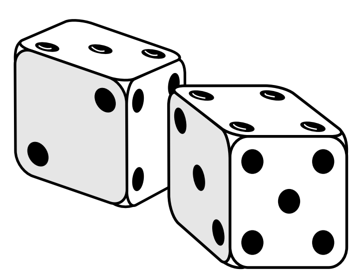
Figure 4 represents sampling without replacement from a small population where successive events are dependent. Figure 5 represents sampling without replacement from a population of unlimited size. Figure 5 also could be used for a small population when sampling with replacements.

Baye's Theorem

Baye's Theorem is a procedure used to revise probabilities based on new empirical information. Probabilities can be revised upward or downward based on the new information. The probability of an event as calculated before doing an empirical study is known as the *prior probability*. For example, let's say prior possibilities of events "E" and "not E" are known. An investigation results in several possible outcomes, each statistically dependent on E. For any particular result (R), the conditional probabilities Pr[R | E] and Pr[R | not E] often are available. Based on the results, Pr[E] and Pr[not E] can be revised upward or downward according to Baye's Theorem (Equation 8). The revised probability is known as a *posterior probability,* which is defined as a conditional probability of the form Pr[E | R], Pr[not E | R].

$$\textit{Eq. 8} \quad Pr[E \mid R] = \frac{Pr[E] \text{ x } Pr[R \mid E]}{(Pr[E] \text{ x } Pr[R \mid E]) + (Pr[\text{not } E] \text{ x } Pr[R \mid \text{not } E])}$$

As an example, a box contains four fair dice and one weighted die that makes the 6 appear two-thirds of the time. If you can't tell the weighted die from the others and you roll a 6, what is the



probability that you tossed the weighted die? Let "E" = weighted die and "not E" = fair die. The empirical information R = the fact that you rolled a six. Prior probabilities are:

Pr[E] = 1/5 (1 weighted die)

Pr[not E] = 4/5 (4 fair dice)

You know that when the weighted die is tossed, the probability of rolling a six is:

Pr[R | E] = 2/3

If a nonweighted die is tossed, the probability of a six is:

Pr[R | not E] = 1/6

Using Baye's Theorem, you can calculate the posterior probability that you rolled the weighted die, given that you rolled a six:

$$\Pr[E \mid R] = \frac{\Pr[E] \times \Pr[R \mid E]}{(\Pr[E] \times \Pr[R \mid E]) + (\Pr[\text{not } E] \times \Pr[R \mid \text{not } E])}$$

$$= \frac{1/5 \times 2/3}{(1/5 \times 2/3) + (4/5 \times 1/6)}$$

$$= \frac{2/15}{2/15 + 4/30}$$

$$= \frac{2/15}{4/15}$$

$$= 1/2$$

Thus, the probability of tossing the weighted die was revised upward from the prior value of ⅕, with no information, to the posterior value of ½, with the information that a 6 was tossed.

Let's look at another example. A wholesale lumber supplier classifies its clients as small, medium, and large depending on how much lumber they purchase on a monthly basis. Overall, 55 percent are small, 35 percent are medium, and 10 percent are large. A new customer is not categorized until after 18 months of doing business with the wholesaler. Management wants to do different amounts of advertising based on each customer's size category and wishes to know whether new customers can be classified after only 3 months of business. Based on company records, 3-month histories for each category were obtained (Table 3).

Table 3.—Percent purchased by clients based on 3 months of purchases.

First 3 months purchases	Percent Small (sm)	 Medium (med)	 Large (L)
<1 MBF	60	15	5
1 MBF–5 MBF	20	45	20
>5 MBF	20	40	75

If a customer purchases less than 1 MBF in the first 3 months, what is the probability that the customer fits in the small category?

$$\Pr[sm \mid <1MBF] = \frac{\Pr[sm] \times \Pr[<1\ MBF \mid sm]}{(\Pr[sm] \times \Pr[<1\ MBF \mid sm]) + (\Pr[med] \times \Pr[<1\ MBF \mid med]) + (\Pr[L] \times \Pr[<1\ MBF \mid L])}$$

$$= \frac{0.55 \times 0.60}{(0.55 \times 0.60) + (0.35 \times 0.15) + (0.10 \times 0.05)}$$

$$= \frac{0.33}{0.388}$$

$$= 0.85$$

The same information can be found using a probability tree (Figure 6):

$$\Pr[sm \mid <1\ MBF] = \frac{P[sm\ and\ <1MBF]}{\Pr[<1\ MBF]}$$

$$= \frac{0.33}{0.005 + 0.330 + 0.0525}$$

$$= 0.85$$

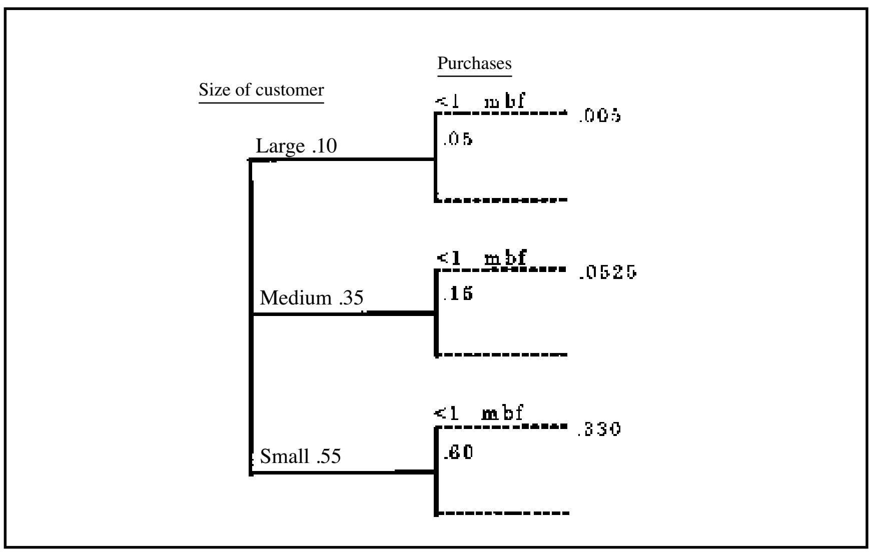


Figure 6.—Partial probability tree depicting size of customer based on amount purchased over a 3-month period (<1 MBF branches).

From the above examples, you can see that it sometimes is easier to solve probability problems by building probability tables or probability trees.

Other publications in this series will expand on the information on modeling and probability theory reviewed in this publication.

For more information

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