Stand Density Management: Results from Research in Southern Pine *

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Introduction

The relationship between stand density and volume growth in southern pine species is a topic of long standing interest. Early work includes that of McClay (1955) who studied the relationship between periodic annual growth per acre and residual stand density after thinning measured as basal area. Using linear regression analysis he found a trend of increasing growth with increasing residual stand density up to a point where growth is maximized. Whether the growth levels decline beyond this point could not be determined conclusively because of a lack of data from high density stands. Similar regression analysis was performed by Wenger et al. (1958) to study the effect of growing-space requirements for thinned and unthinned natural stands of loblolly pine for a 5-year period. In thinned stands the relationship between volume growth and density varied with site quality, but not with stand age. Alternatively, both variables were significant for unthinned stands.

Nelson et al. (1961, 1963) used the 5-year period following Wenger et al. (1958), to again develop separate equations for thinned and unthinned stands in relation to age, site and stand density. This study did not discover differences in volume increment between thinned and unthinned conditions. The relationships obtained using the regression equations indicated that production in thinned stands increases with stand density on better sites and reaches an optimum on poor sites. Recently, Zeide (2004) developed and fit, for loblolly pine, a theoretical equation for determining the ‘optimal stand density’. He concluded that maximum growth occurs at maximum stand density, supporting the observations reported by Curtis et al. (1997) for Douglas-fir.

Results from region-wide thinning study in Loblolly Pine

Data from a region-wide thinning study that was installed and maintained by the Loblolly Pine Growth and Yield Research Cooperative were used to (a) model the general pattern between gross and net total volume increment and stand density, (b) develop an equation for estimating expected gross and net total volume periodic increment (PAI) in relation to site quality and stand density, and (c) develop an equation for including the effect of merchantability limits, defined by threshold diameter and top diameter, on expected gross and net PAI.

The data consisted of plot measurements first collected in the dormant seasons of 1980-1982 in stands of different ages, site indexes and densities on cutover, site prepared areas. A total of 186 locations were selected from the Piedmont and Coastal Plain physiographic regions. Three plots with similar spacing, basal area and site index were established at each location. Each plot was randomly assigned to a; light thinning, heavy thinning or control (no thinning) treatment. The light thinning treatment removed approximately 1/3 and the heavy thinning approximately 1/2 of the basal area. After installation, each plot was re-measured at 3 years intervals (Burkhart et al. 1985).

For this study the plots used were those where: (a) the planted loblolly pine represented more than 95% percent of the total basal area at the first measurement after thinning, (b) only plots with a stand age between 10 and 20 years and (c) only plots with complete and consistent growth information for a 9-year growth period. A total number of 206 plots situated at 85 locations met these criteria and were selected for subsequent data analysis.

Total PAI and stand density relationship

A regression analysis approach was used for examining the relationship between volume increment and stand density:

\[ \text{PAI} = a_0 \cdot BA_i \cdot \exp(a_2 \cdot BA), \]

where \( \text{PAI} \) = periodic annual volume increment (m\(^3\) ha\(^{-1}\) year\(^{-1}\)), \( BA \) = basal area (m\(^2\) ha\(^{-1}\)) at the beginning of the growth period and \( a_i \) = parameters to be estimated (\( i = 0, 1, \) and 2).
The relationship between total volume increment and basal area was examined for a range of stand ages and site index classes. Three site index classes were defined as poor (SI < 16.8m, based age 25 years), medium (16.8 < SI < 19.8) and high (SI > 19.8m) productivity.

Because of its flexibility, it is possible to test whether the data empirically support an optimum or increasing relationship between volume $PAI$ and basal area. An optimum relationship means that there exists a level of basal area where a maximum volume increment occurs. An increasing relationship exists when the volume increment increases with basal area therefore the maximum volume increment occurs at a maximum level of basal area. If parameter $a_2$ is found to not be significant then there is evidence against the hypothesis of an “optimum”, and an increasing pattern is present. If all parameters are significant then there is evidence of an optimum pattern between volume increment and basal area.

Equation (1) was fitted to both gross and net, $PAI$ for total volume, for each of the three site index classes using non-linear regression methods. The curves generated from fitting equation (1) are presented in Figure 1. Figure 1 shows that site index has an effect on the levels of gross and net $PAI$. After testing the significance of the parameters it can be concluded that gross $PAI$ indicates in all cases an increasing pattern. Net $PAI$ in total volume presents an optimum pattern for high and medium site index classes. The basal area that maximizes net $PAI$ seems to be related to site index classes.

**Modeling gross and net PAI in total volume**

Based on the generalized exponential and power function (1) a single equation was developed for estimating both gross and net total volume increment dependent on stand density ($BA$) and site quality ($SI$). The final equation has the following form:

$$PAI = SI^{b_1} \exp(b_1 / BA) \exp(b_2 (1 / SI) BA^2 I),$$

where $SI =$ site index (m) and $b_i=$parameters to be estimated ($i=0,1,$ and 2).
This equation includes an indicator function \( I \), which takes the value of zero if an estimate of gross PAI is desired or one for an estimate of net PAI. It has the flexibility of modeling an increasing pattern for gross increment and an optimum pattern for net increment.

According this equation maximum net PAI occurs on better sites at higher levels of stand density relative to lower site index classes. Equation (2) was fitted to the data and the estimated parameters were \( b_0=1.0370, b_1=-6.2875 \) and \( b_2=-0.00347 \). All the parameters were significant at \( \alpha=0.05 \) and the equation explained 54% of the total variability (RMSE=2.98 m³ ha⁻¹ year⁻¹). Figure 2 shows the curves generated by equation (2) for site indexes of 15, 20 and 25 m.

Our observations and equation for gross and net PAI for total volume agree with Nyland (2002, p.404), who reported that gross increment must always be greater than net increment and it is expected that the net increment will reach a maximum at some intermediate stand density.

**The effect of merchantability limits**

The definition used for merchantability has been mentioned as one of the most important limitations to obtaining general conclusions on the growth-density relationship (Clutter et al. 1983, Curtis et al. 1997, Zeide 2001). Amateis et al. (1986) indicate that the merchantability limits for stands are dependent on both top diameter (\( t \)) and threshold diameter (\( d \)). Therefore, these two variables were included in a system of equations for estimating, in a more flexible way, expected gross and net PAI. Using the same database, gross and net PAI for different threshold and top diameter combinations was computed. A range of threshold diameters was defined according to Amateis et al. (1986) from \( d=9.14 \) to 24.38 cm and a range of top diameter from \( t=5.08 \) to 20.32 cm both by steps of 2.54cm. The basic structure for developing the system of equations comes from equation (2). First, an equation for estimating gross PAI to any merchantable portion (\( PAI_{G,m} \)) was developed

\[
(3) \quad PAI_{G,m} = SI^{c_0} \exp \left( \frac{c_1}{BA} + c_2 \left[ \frac{d}{D} \right]^{c_1} + \left( \frac{t}{D} \right)^{c_2} \right),
\]

where \( D \) is the quadratic mean diameter at breast height (o.b), in cm.
This equation is conditioned such that when both the threshold diameter \((d)\) and the top diameter \((t)\) are equal to zero it produces an estimation of gross total \(PAI\). When \(t=0\) then the merchantable portion is determined by the threshold diameter and when \(d=0\) the merchantable portion is determined by the top diameter. The parameters estimated were \(c_0=1.0486, c_1=-4.7157, c_2=-0.0242, c_3=7.5782\) and \(c_4=12.1793\). All the parameters were significant at \(\alpha=0.05\) and this equation explained 70% of the total variability (RMSE=3.7 m\(^3\) ha\(^{-1}\) year\(^{-1}\)). A second equation for estimating net \(PAI\) for any merchantable portion of the stand \((PAI_{N,m})\) was developed:

\[
(PAI_{N,m}) = PAI_{G,m} \exp\left\{ c_5 \left( \frac{1}{SI} \right) BA^2 \left( \frac{d+1}{D} \right) \right\},
\]

where the exponent reduces \(PAI_{G,m}\) for estimating \(PAI_{N,m}\). The estimated parameters for equation (4) were \(c_5=-0.00060\) and \(c_6=-2.5732\). All parameters were significant at \(\alpha=0.05\) and the equation explained 99% of the total variability in \(PAI_{N,m}\) based on \(PAI_{G,m}\) (RMSE=0.3686 m\(^3\) ha\(^{-1}\) year\(^{-1}\)).

**Conclusion**

The analysis performed indicated that the relationship between gross \(PAI\) in total volume and basal area for all site index classes follows an increasing pattern. It was established that in general, net \(PAI\) in total volume follows an optimum pattern. However, an increasing pattern was observed empirically for the poor site index class. The general relationship between gross and net increment found here agree with the descriptions given by Nyland (2002, p.404), where gross and net follow an increasing and optimum pattern, respectively.

The effect of stand density expressed as basal area and site quality given as site index was included in a single equation, maintaining the general relationship existing between gross and net increment (Figure 2). Although the equation explained only 54% of the total variability in \(PAI\), it represents a consistent and general quantitative expression of expected gross and net increment in total volume for the range of stand ages and site index classes studied.
The system of equations developed for including the effect of merchantability limits in terms of threshold and top diameter seems to be adequate and flexible enough for studying the volume increment and basal area relationship for a wide variety of stand conditions.

**Literature Cited**


Figure 1. Trend of equation (1) fitted to gross (circles) and net (dots), periodic annual increment ($PAI$) for (a) high, (b) medium and (c) poor site index classes.
Figure 2. Estimated gross and net periodic annual increment (PAI o.b.) for total volume of planted loblolly pine trees using equation (3) for site indexes 15, 20 and 25 m.