Performance Excellence in the Wood Products Industry
Statistical Process Control

Part 9: Process Capability Analysis

EM 9111 • May 2015

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Our focus for the prior publications in this series has been on introducing you to Statistical Process Control (SPC)—what it is, how and why it works, and how to use various tools to determine where to focus initial efforts to use SPC in your company.

Through this series, we described a variety of quality tools in terms of how they support implementation and use of SPC. The two primary SPC tools are control charts and process capability analysis.

A quick recap of this publication series: Parts 1 and 2 introduced SPC: what it is and how it works. In Parts 3 through 8, we walked through an example from XYZ Forest Products Inc. (a fictional company), following along as the company's quality improvement team began using SPC in response to customer complaints about size of wooden handles being out of specification (hereafter called out-of-spec handles). We described how the team:

- Used Pareto charts and check sheets to decide where to focus efforts (Part 3)
- Constructed flowcharts to build consensus on the steps involved and help define where quality problems might be occurring (Part 4).
- Created cause-and-effect diagrams to identify potential causes of a problem (Part 5)
- Designed an experiment to hone in on the true cause of the problem (Part 6)
- Used the primary SPC tools—variables and attributes control charts—to first assess stability of the process and then monitor key variables to ensure the process remains stable and predictable over time (Parts 7 and 8)

It is important to not lose sight of the primary goal of SPC: Improve quality, and in so doing, improve customer satisfaction and the company's profitability.

We've spent considerable effort identifying XYZ's most important quality problems, determining how to solve them, and then ensuring the process remains stable. Now we need to step back and ask how well the new-and-improved process is able to meet customer expectations. In short, we must shift our focus from stability to capability.

Our focus on customers in process capability analysis is not limited to external customers. Internal customers (that is, processes down the line) are important, too. In our XYZ example, external customer expectations are related to the size of wooden handles. However, the designed experiment (Part 6) revealed that controlling moisture content is a means of controlling handle dimensions. In that way, moisture content is a specification established for an internal customer.

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Does stability equal capability?

In Part 7, we developed control charts for data on wood moisture content. With evidence that the process is **in control** (stable), we can have confidence that our estimates of process centering and variability are reliable. We have reasonable assurance that the process is stable. But does that mean the process is capable of meeting customer expectations? Unfortunately, the answer is no.

Stability reflects how the process is performing; **capability** reflects the customer's desires for the products. Said another way, stability is related to the "voice of the process" and capability is related to the "voice of the customer." The two are generally unrelated.

For example, a customer may specify that moisture content must be an average of 6% ± 1%. To give an extreme example, if we gather data, construct a variables control chart (see Part 7), and find our process is stable at an average moisture content of 10% ± 5%, we have assurance of stability. But clearly the process is not capable. It is centered at far too high of a moisture content with far too much variability to meet the specifications.

But how bad is it? How much defective product can we expect to produce, and what does that cost the company? How much could we reduce the defect rate, and thus the costs, if we were able to reduce variability from 5% to 3%? Process capability analysis enables us to answer these questions.

Process capability analysis

Continuing with our XYZ example, remember that discussion in *Part 7: Variables Control Charts* ended with the team assuming that the process exhibited control at an average moisture content of 6.5% and an average range of 2.6%. Can the team now be confident that the size-out-of-spec problem will go away? No. A stable process is not necessarily capable.

In fact, the goal of the designed experiment (Part 6) was to control moisture content at 6%. The process is already slightly off target. But what is the impact of being slightly off target? Is it worth investing the effort to bring the process on target at 6%? Also, is an average range of 2.6% acceptable? What is an acceptable level of variability?

We now know we can control moisture content at 6.5% with a range of 2.6%. Using the $\overline{R}/d_2$ formula presented in Part 7, we can estimate the process standard deviation. Since d_2 (a table value) for samples of size five is 2.326, our estimate of standard deviation for this process is about 1.12% (2.6/2.326). And given what we know about the normal distribution, we can expect that more than 99% of moisture content readings should fall between the mean plus and minus three standard deviations (6.5 ± 3×1.12, or 6.5 ± 3.4), which is from about 3% to 10% moisture content.

So to determine the acceptable level of variability, we could conduct another designed experiment with those moisture content values for birch and poplar and measure the number of out-of-spec handles, as before. If the number of out-of-spec handles is acceptable, we would continue to monitor the process to ensure it stays at these $\overline{\overline{x}}$ and $\overline{R}$ values.

Why is process capability analysis important?

Some customers require suppliers to meet or exceed specific process capability indices.

For example, a customer might say they require suppliers to provide evidence that their capability index for a certain parameter (e.g., moisture content, thickness) was at least 1.33. In such cases, you must meet these criteria or lose a customer. Suddenly, you are motivated to learn how to calculate and interpret capability indices!

However, for this discussion, let's assume the experiment indicates this target value and variability are too high and result in excessive out-of-spec product. Further, the experiment confirms that we need to maintain specifications of 4% to 8% (6% ± 2%) moisture content. To describe the current capability of the process, we need some simple way to compare the variability of our process relative to the specifications. Process capability analysis does just that with a few simple ratios.

Process capability: A quick visual explanation

Figure 1 shows normal curves for two process distributions. LSL and USL are the lower specification limit and upper specification limit, respectively. The lower, wider distribution is the curve for XYZ's current process (average of 6.5% ± 3.4% moisture content)[1]. The taller, narrower distribution represents the specifications (centered at 6% ± 2% moisture content). The shaded area represents **defects** (material beyond the specifications).

From this graph, we can see that by being off target and having excessive variability, XYZ is producing a significant amount of material beyond the specifications, specifically the upper specification. A graph like this can help illustrate the concepts, but from a practical standpoint, we need to quantify the relationship between the spread of the process and the spread of the specifications. We can do this by using capability indices.

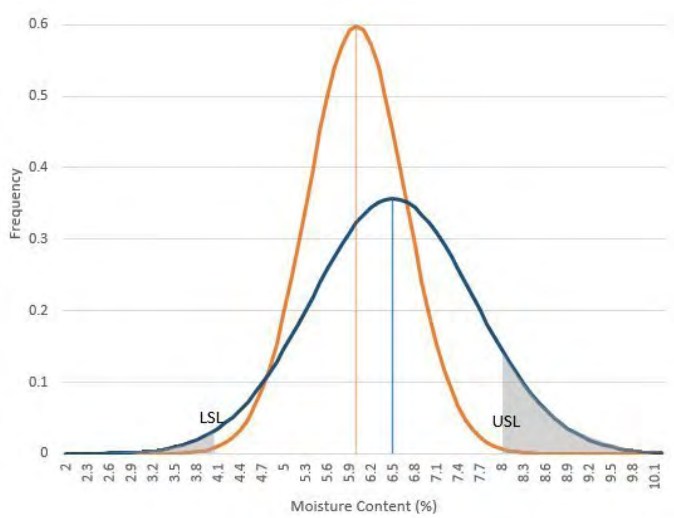

Figure 1. Effect of process standard deviation and centering on the defect rate.

[1] Recall that the average range ($\bar{R}$) was 2.6%. For samples of size five, this equates to a process standard deviation of 1.12 ($\bar{R}/d_2$, where d_2 is a table value that varies with sample size). Therefore, the distribution is from the mean plus and minus three standard deviations, or 6.5% ± 3.4% moisture content.

Capability indices

The first capability index we will discuss is C_p, which is calculated as:

$$C_p = \frac{USL - LSL}{6\hat{\sigma}}$$

where σ is the process standard deviation and the ^ (hat or caret) symbol over it means “estimate.” Recall that σ is the true standard deviation for a normally distributed variable, and generally the best we can do is estimate it by sampling the process. The value $6\hat{\sigma}$ is the total width of process variability (that is, plus and minus three standard deviations). Therefore, C_p is a ratio of the specification width to total process width.

Given that our specification for moisture content is 6% ± 2%, the LSL is 4% and the USL is 8%. Because the estimate of $\overline{R}$ was 2.6% and the table value for d_2 is 2.326, $\hat{\sigma}$ is 1.12 (2.6/2.326) and C_p is:

$$C_p = \frac{USL - LSL}{6\hat{\sigma}} = \frac{8 - 4}{6 * 1.12} = 0.60$$

What does this mean? In simple terms, the higher the C_p value the better. A C_p equal to 1.0 means we are exactly on the specifications. A C_p less than 1.0 indicates that process variability is higher than the specification width, and therefore producing a substantial amount of defective material. Later in this publication, we’ll be more precise about how much defective material is produced. In the XYZ example, C_p is less than 1.0 so we know that process variability is too high and, therefore, the process is not capable of meeting the specifications.

C_p is a simple ratio that is relatively easy to calculate. Its primary limitation is that it does not account for process centering relative to the target. Theoretically, a manufacturing process could be centered far away from the target, perhaps even producing 100% defective product. Yet if the process variability were low, C_p would indicate everything was okay.

To account for process variability and centering relative to the target, we use another process capability index, C_{pk}. The formula for C_{pk} is:

$$C_{pl} = \frac{(\hat{\mu} - LSL)}{3\hat{\sigma}}, C_{pu} = \frac{(USL - \hat{\mu})}{3\hat{\sigma}}$$
$$C_{pk} = \min\,(C_{pl}, C_{pu})$$

where C_{pl} and C_{pu} are the lower and upper process capability indices, respectively, relative to the process average; $\hat{\mu}$ is our estimate of the process average; and min indicates that C_{pk} is the minimum (lesser or worst-case) of Cpl and C_{pu}.

To calculate C_{pk}, we use $\hat{\sigma}$ as calculated above, and we use $\overline{\overline{x}}$ as $\hat{\mu}$. Because our estimate of $\overline{\overline{x}}$ is 6.5% moisture content, C_{pk} is then:

$$C_{pl} = \frac{(6.5 - 4)}{3 * 1.12} = 0.74, C_{pu} = \frac{(8 - 6.5)}{3 * 1.12} = 0.45$$
$$C_{pk} = \min(0.74, 0.45) = 0.45$$

C_{pk} is interpreted much the same as C_p, Below 1.0 is bad and above 1.0 is good. However, C_{pk} provides a bit more information. If C_{pl} and C_{pu} are equal, we know the process is on target. And when the process is on target, C_p and C_{pk} are the same.

In this case, because C_{pl} and C_{pu} are not equal, we know the process is off center. Specifically, because C_{pu} is lower than C_{pl}, we know the process is centered too close to the USL. Therefore, excessive defects are produced above the USL, as seen in Figure 1. We can use the normal distribution to estimate the percentage of defective product (see Estimating defect rates, below).

What if we adjust the process to put it on target? In other words, what if we shift the process center from 6.5% to 6%? Because C_p does not account for process centering, it would not be affected. C_{pk}, however, would increase from 0.45 to 0.60. Remember, C_p and C_{pk} are the same when a process is on center.

Perhaps more importantly, what if we also engage in continuous process improvement activities and reduce the average range ($\overline{R}$) from 2.6 to 2.0? Now our estimate of C_{pk} (and C_p) would be 0.77. Because it is still below 1.0, we might ask: What does $\overline{R}$ need to be to attain a C_{pk} of 1.0? We need a process that operates in control at an average range ($\overline{R}$) of about 1.55 to achieve a C_{pk} of 1.0 for this process.

Some customers stipulate C_p or C_{pk} values for their suppliers. However, for internal customers, you might find it difficult to generate much excitement related to increasing a process capability index from 0.45 to 1.0. After all, the language of business is money and these indices don't directly give any idea of what this means in terms of defect rates and, therefore, the cost of poor quality. For this, we need to delve a bit more into statistics.

Estimating defect rates

What are the practical implications of these capability indices? For example, if we move from a C_{pk} of 0.45 to 0.77, what does that mean for the company's defect rate, costs, and profitability? This is an advanced, but critical, topic.

To estimate the defect rate, we must know something about how the data are distributed. In this case, we need to know how the moisture content data are distributed. For example, if we collect a few hundred readings and plot a histogram (see Part 2), would the data appear to be normally distributed as shown in Figure 1? Or would the data be skewed such that most values are on the lower end and there is a long tail out to the right for higher moisture content values?

Unless you have a statistician on staff (or happen to be one yourself!), it can become very complex to determine the distribution that best fits the data and then use that information to estimate defect rates. Therefore, the most common approach is to simply *assume* the data are normally distributed. All we need to draw normal curves is the mean and standard deviation—and we have estimates of those values from our control charts.

Let's return to the XYZ example. Given that the process is operating at an average moisture content of 6.5% ± 3.4% (average range of 2.6%), C_{pk} is 0.45. However, the specifications are 6% ± 2%. In Figure 1, the shaded area beyond the USL (8%) is fairly large. But how large? If we assume moisture content is normally distributed, what percentage of handles can we expect to have a moisture content above 8%?

To obtain this estimate, we again turn to tables that are typically found in the appendix of quality control textbooks. Look for a table of **area under the standard normal curve**. What does standard normal curve mean? Since there are infinite combinations of means and standard deviations, we would need an infinite number of tables of normal distributions. It's not possible to list them all in a textbook. Therefore, we must standardize our data so it can be represented by a normal distribution with a mean of zero and standard deviation of one. To do this, we calculate a **z statistic**:

$$z = \frac{X - \hat{\mu}}{\hat{\sigma}}$$

where X is the value of interest, and $\hat{\mu}$ and $\hat{\sigma}$ are, as before, estimates of the mean and standard deviation of the process. The z value tells us how many standard deviations above the average a value is.

For example, to calculate the percentage of handles that will have a moisture content greater than 8% (the shaded area on the right of Figure 1), we calculate the z value as:

$$z = \frac{8 - 6.5}{1.12} = 1.34$$

Then we refer to a table of areas under the standard normal curve and look up the value for 1.34[2], which is 0.9099. This value means that 90.99% of the values will be *below* 1.34 on the standard normal curve, which is 8% moisture content in our example. The table lists the area to the left of the value (below).

However, we're interested in the area to the right—the values greater than 8%. Since the area under the normal curve is 1.0, simply subtract the table value from 1. In this case, the result is 1 - 0.9099, or 0.09. So we expect about 9% of the moisture content readings to be greater than the USL of 8%.

Is this the estimate of the defect rate? Not quite. Remember: There is also a lower specification. To get that value, using the LSL of 4%, we estimate z as:

$$z = \frac{4 - 6.5}{1.12} = -2.23$$

Because we are interested in the area (percentage) to the left of this value, we can read the value directly from the table as 0.013, or about 1.3%.

Therefore, the total estimate of defects is 9% + 1.3%, or approximately 10.3%.

[2] You can do this in spreadsheet software as well. For the example here, you would enter =NORMDIST(8, 6.5, 1.12, 1). The final "1" in the formula is to get the cumulative value; that is, the area under the normal curve.

Cost of defects

What does this defect rate mean in terms of cost? To know that, we need to connect moisture content to the costs of rejecting or reworking handles. In Part 3, we learned that XYZ has an average scrap and rework cost of $12 for out-of-spec handles. Again, without hard data, we'll have to make some assumptions. For this example, we'll assume that moisture content beyond the specifications results in out-of-spec handles (as demonstrated in the designed experiment).

If XYZ operates 5 days per week, that equates to 21.67 days per month. If the company produces 5,000 handles per day, that is approximately 108,000 handles per month. At the 10.3% defect rate, about 11,000 handles would need to be scrapped or reworked every month. At an average scrap/rework cost of $12 each, the total is just over $130,000 per month.

This is the estimated defect rate and accompanying cost given an average moisture content of 6.5% ± 3.4% (average range of 2.6%), which equates to a C_{pk} of 0.45.

What if the XYZ team shifts the process to be on target at 6% moisture content with an average range ($\overline{R}$) of 2.0, so that the new C_{pk} is 0.77? (Note: Of course, the team could aim for a C_{pk} of 1.0 or even 2.0. However, there's wisdom in setting realistic goals for initial quality improvement efforts, and then trying to exceed those goals. As the saying goes, under-promise and over-deliver).

With an $\overline{R}$ of 2.0, the estimate of standard deviation is now 0.86 (2.0/2.326). And z values would be:

$$z = \frac{4-6}{0.86} = -2.33 \text{ and } z = \frac{8-6}{0.86} = 2.33$$

The table values for areas under the normal curve for these z values are about 0.01 each, so the new defect rate is about 2%. Monthly scrap/rework costs would be about $26,000. That's a reduction in cost of more than $100,000 per month!

What if XYZ further improves the process to be on target at a C_{pk} of 1.0; that is, at 6% moisture content with an $\overline{R}$ of 1.55 (standard deviation of 0.67)? The defect rate would drop to 0.3% (that's 3 in 1,000 vs. the 1 in 10 we have now), and the monthly costs would be about $3500.

SPC: Summary

If you want to convince managers at your company to invest in quality training so you can effectively pursue quality improvement, we recommend following the process outlined in this publication series:

- Parts 1 and 2: Introduce company personnel to SPC: what it is and how and why it works.
- Part 3: Talk with customers about quality issues. Use Pareto charts and check sheets to decide where to focus initial quality improvement efforts. Devote some time to determine what the specific defect categories cost in terms of reject from customers, or internally in terms of scrap, rework, and downgrade.
- Part 4: Construct flowcharts to build consensus on the steps involved and help define where in the process quality problems might be occurring.
- Part 5: Create cause-and-effect diagrams to identify potential root causes of the top quality issues.
- Part 6: Conduct designed experiments to hone in on the true cause of the problem and understand how key process variables (e.g., moisture content, species, tooling) affect quality. Use designed experiments to establish optimal process settings.
- Parts 7 and 8: Use control charts to determine if the process is stable. When charts demonstrate stability, estimate centering and variability of the process.
- Part 9: Use process capability analysis to assess whether the process is capable of meeting specifications. Estimate potential savings due to quality improvement.

The bottom line is, well, still the bottom line. You are unlikely to find many managers who will get excited when you request funds to conduct quality improvement projects with a goal of reducing the standard deviation from 1.12 to 0.86 or increasing the C_{pk} from 0.45 to 0.77. These values are important, but you need to connect them to costs and profitability. Managers will likely pay attention if you are able to explain (and document) that a quality improvement project will reduce costs by $100,000 per month due to a reduced scrap/rework rate, as in the XYZ example.

Of course, you'll have to be prepared to justify your cost savings estimates. This publication series provides information you can use to prepare to present concepts backed with data and facts. Spend time quantifying what specific types of defects cost the company. In the wood products industry, we sometimes assume that scrap, rework, and downgrade are insignificant given the relatively low cost of small wood components. However, when you consider the volume produced in a day, it doesn't take long for even relatively low defect rates to result in significant costs to the company, as we've tried to demonstrate here.

And finally, remember this advice: "Total quality management is a journey, not a destination." (Thomas H. Berry, 1990. *Managing the Total Quality Transformatio*n).

Quality improvement is a continuous pursuit that requires proactively seeking customer input, improving processes, and then repeating those steps in an effort to reduce variability and thereby improve quality, improve customer satisfaction, and improve profitability.

For more information

The Oregon Wood Innovation Center website provides common table values for SPC: *http://owic.oregonstate.edu/spc*

The listing for the SPC Part 7 publication (EM 9109) in the OSU Extension Catalog also includes a supplemental spreadsheet file that includes data and a chart for capability analysis (see tab labeled “capability”): *https://catalog.extension.oregonstate.edu/em9109*

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Published May 2015.